CBSE 2025 EXAMINATIONS
XII MATHEMATICS (041)
Series ZWY7X ♦ Q.P. Code 65/7/1; 65/7/2; 65/7/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

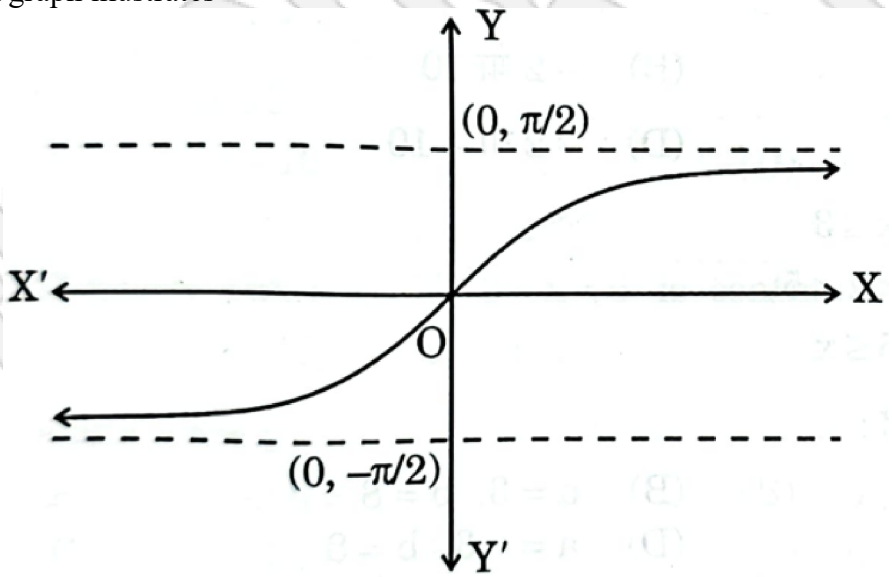
   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/7/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. The given graph illustrates



(a) $y = \tan^{-1} x$ (b) $y = \operatorname{cosec}^{-1} x$ (c) $y = \cot^{-1} x$ (d) $y = \sec^{-1} x$

Sol. (a) The graph illustrates $y = \tan^{-1} x$.

02. Domain of $f(x) = \cos^{-1} x + \sin x$ is

(a) $\mathbb{R}$ (b) $(-1, 1)$ (c) $[-1, 1]$ (d) ϕ

Sol. (c) $\cos^{-1} x$ is defined when $x \in [-1, 1]$, and $\sin x$ is define when $x \in \mathbb{R}$.

So, domain of $f(x) = x \in [-1, 1] \cap \mathbb{R} = x \in [-1, 1]$.

03. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(a) 9 (b) 512 (c) 615 (d) 64

Sol. (b) Total number of elements in a matrix of order 3×3 is 9.
So, total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$ $=2^9=512$.

04. The matrix $A=\begin{bmatrix}\sqrt{3} & 0 & 0\\ 0 & \sqrt{2} & 0\\ 0 & 0 & \sqrt{5}\end{bmatrix}$ is a/an

(a) scalar matrix (b) identity matrix (c) null matrix (d) symmetric matrix

Sol. (d) A is a symmetric matrix, as $A=A^T$.

05. If A and B are two square matrices each of order 3 with $|A|=3$ and $|B|=5$, then $|2AB|$ is

(a) 30 (b) 120 (c) 15 (d) 225

Sol. (b) $|2AB|=2^3|AB|=8\times|A||B|=8\times3\times5=120$.

06. Let A be a square matrix of order 3. If $|A|=5$, then $|\text{adj } A|$ is

(a) 5 (b) 125 (c) 25 (d) -5

Sol. (c) $|\text{adj } A|=|A|^{3-1}=5^2=25$.

07. If $\begin{bmatrix}2x-1 & 3x\\ 0 & y^2-1\end{bmatrix}=\begin{bmatrix}x+3 & 12\\ 0 & 35\end{bmatrix}$, then the value of $(x-y)$ is

(a) 2 or 10 (b) -2 or 10 (c) 2 or -10 (d) -2 or -10

Sol. (b) On comparing the corresponding elements, $3x=12\Rightarrow x=4$,
$y^2-1=35 \Rightarrow y^2=36 \therefore y=\pm6$
Now $(x-y)=4-(\pm6)=-2, 10$.

08. If $f(x)=\begin{cases}1, \text{ if } x\le3\\ ax+b, \text{ if } 3<x<5\\ 7, \text{ if } 5\le x\end{cases}$ is continuous in $\mathbb{R}$, then the values of a and b are

(a) $a=3, b=-8$ (b) $a=3, b=8$ (c) $a=-3, b=-8$ (d) $a=-3, b=8$

Sol. (a) As f is continuous in $\mathbb{R}$, so it shall be continuous at $x=3$ and $x=5$.
Now $\lim\limits_{x\to3^-}f(x)=\lim\limits_{x\to3^+}f(x)=f(3)$ and $\lim\limits_{x\to5^-}f(x)=\lim\limits_{x\to5^+}f(x)=f(5)$
$\Rightarrow \lim\limits_{x\to3^+}(ax+b)=1$ and $\lim\limits_{x\to5^-}(ax+b)=7$
$\Rightarrow 3a+b=1$ and $5a+b=7$
Solving these equations, we get $a=3, b=-8$.

09. If $f(x)=-2x^8$, then the correct statement is

(a) $f'\left(\frac{1}{2}\right)=f'\left(-\frac{1}{2}\right)$ (b) $f'\left(\frac{1}{2}\right)=-f'\left(-\frac{1}{2}\right)$

(c) $-f'\left(\frac{1}{2}\right)=f\left(-\frac{1}{2}\right)$ (d) $f\left(\frac{1}{2}\right)=-f\left(-\frac{1}{2}\right)$

Sol. (b) Note that, $f'(x)=-16x^7$.

As $f'\left(\frac{1}{2}\right)=-16\left(\frac{1}{2}\right)^7=-\frac{1}{2^3}$; $f'\left(-\frac{1}{2}\right)=-16\left(-\frac{1}{2}\right)^7=\frac{1}{2^3}$.

Clearly, $f'\left(\frac{1}{2}\right)=-f'\left(-\frac{1}{2}\right)$.

10. A spherical ball has a variable diameter $\frac{5}{2}(3x+1)$. The rate of change of its volume w.r.t. x, when $x=1$, is

(a) 225π (b) 300π (c) 375π (d) 125π

Sol. (c) Volume, $V=\frac{4\pi}{3}\times\left[\frac{5}{4}(3x+1)\right]^3=\frac{4\pi}{3}\times\frac{125}{64}\times(3x+1)^3$

$\therefore \frac{dV}{dx}=\frac{\pi}{3}\times\frac{125}{16}\times3\times3(3x+1)^2$

So, $\left(\frac{dV}{dx}\right)_{\text{at } x=1}=\frac{125\pi}{16}\times3(3\times1+1)^2=375\pi$ units^3/units.

11. If $f:\mathbb{R}\to\mathbb{R}$ is defined as $f(x)=2x-\sin x$, then f is

(a) a decreasing function (b) an increasing function

(c) maximum at $x=\frac{\pi}{2}$ (d) maximum at $x=0$

Sol. (b) As $f'(x)=2-\cos x>0\ \forall\, x\in\mathbb{R}$, so f is an increasing function.

Recall that, $-1\le\cos x\le1\ \forall\, x\in\mathbb{R}$.

12. $\int\frac{e^{9\log x}-e^{8\log x}}{e^{6\log x}-e^{5\log x}}dx$ is equal to

(a) $x+C$ (b) $\frac{x^2}{2}+C$ (c) $\frac{x^4}{4}+C$ (d) $\frac{x^3}{3}+C$

Sol. (c) $\int\frac{e^{9\log x}-e^{8\log x}}{e^{6\log x}-e^{5\log x}}dx=\int\frac{x^9-x^8}{x^6-x^5}dx=\int\frac{x^8(x-1)}{x^5(x-1)}dx=\int x^3\,dx=\frac{x^4}{4}+C$.

13. For a function $f(x)$, which of the following holds true?

(a) $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$

(b) $\int_{-a}^{a} f(x)\,dx=0$, if f is an even function

(c) $\int_{-a}^{a} f(x)\,dx=2\int_0^a f(x)\,dx$, if f is an odd function

(d) $\int_0^{2a} f(x)\,dx=\int_0^a f(x)\,dx-\int_0^a f(2a+x)\,dx$

Sol. (a) $\int_a^b f(x)\,dx=\int_a^b f(a+b-x)\,dx$ holds true for a function $f(x)$.

14. $\int\frac{e^x}{\sqrt{4-e^{2x}}}dx$ is equal to

(a) $\frac{1}{2}\cos^{-1}(e^x)+C$ (b) $\frac{1}{2}\sin^{-1}(e^x)+C$ (c) $\frac{e^x}{2}+C$ (d) $\sin^{-1}\left(\frac{e^x}{2}\right)+C$

Sol. (d) Put $e^x=t\Rightarrow e^x dx=dt$.

So, $\int\frac{e^x}{\sqrt{4-e^{2x}}}dx=\int\frac{1}{\sqrt{4-t^2}}dt=\sin^{-1}\frac{t}{2}+C=\sin^{-1}\left(\frac{e^x}{2}\right)+C$.

15. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i}+15\hat{j}+6\hat{k}$ and the other is along the vector $2\hat{i}+10\hat{j}+\lambda\hat{k}$, then the value of λ is

(a) 6 (b) 1 (c) $\frac{1}{4}$ (d) 4

Sol. (d) The direction ratios for parallel vectors are proportional.

So, $\frac{3}{2}=\frac{15}{10}=\frac{6}{\lambda}$. That is, $\frac{3}{2}=\frac{6}{\lambda} \quad \Rightarrow \lambda = 4$.

16. If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are

(a) orthogonal vectors (b) parallel to each other
(c) unit vectors (d) collinear vectors

Sol. (a) Consider $|\vec{a}+\vec{b}|^2=|\vec{a}-\vec{b}|^2$

That is, $|\vec{a}|^2+|\vec{b}|^2+2\vec{a}.\vec{b}=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}.\vec{b}$

$\Rightarrow 4\vec{a}.\vec{b}=0 \quad \therefore \vec{a}\perp\vec{b}$.

Hence, $\vec{a}$ and $\vec{b}$ are orthogonal vectors.

17. If $P(A)=\frac{1}{7}$, $P(B)=\frac{5}{7}$ and $P(A\cap B)=\frac{4}{7}$, then $P(\bar{A}\,|\,B)$ is

(a) $\frac{6}{7}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{1}{5}$

Sol. (d) $P(\bar{A}\,|\,B)=\frac{P(\bar{A}\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}$

$\Rightarrow P(\bar{A}\,|\,B)=\frac{\frac{5}{7}-\frac{4}{7}}{\frac{5}{7}}=\frac{1}{5}$.

Remark Here $P(A\cap B) > P(A)$, which is not possible.

18. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is

(a) $\frac{2}{13}$ (b) $\frac{3}{26}$ (c) $\frac{19}{26}$ (d) $\frac{3}{13}$

Sol. (b) Required probability $=\frac{1}{2}\times\frac{12}{52}=\frac{3}{26}$.

Direction : *Question numbers* ***19*** *and* ***20*** *are Assertion (A) and Reason (R) based questions carrying* ***1 mark*** *each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ is continuous at $x=0$.

Reason (R) : When $x\to 0$, $\sin\frac{1}{x}$ is a finite value between -1 and 1.

Sol. (b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).

Note that, $\lim\limits_{x\to 0}\left(x\sin\frac{1}{x}\right)=0\times\sin\frac{1}{0}=0=f(0)$ so, f is continuous at $x=0$.

Remark Statement in Reason (R) is **not completely** explaining the statement in Assertion (A).

20. **Assertion (A) :** Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1}x$ is defined for $x\in\mathbb{R}-(-1,1)$.

Sol. (a) Note that, $\frac{\sqrt{3}}{2}\in(-1,1)$. So, Assertion (A) is true, Reason (R) is also true and Reason (R) is the correct explanation of Assertion (A).

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. Let $f:A\to B$ be defined by $f(x)=\frac{x-2}{x-3}$, where $A=\mathbb{R}-\{3\}$ and $B=\mathbb{R}-\{1\}$. Discuss the bijectivity of the function.

Sol. Given $A=\mathbb{R}-\{3\}$, $B=\mathbb{R}-\{1\}$. Let $f:A\to B$ be defined by $f(x)=\frac{x-2}{x-3}\ \forall x\in A$.

Let $x_1,x_2\in A$ and $f(x_1)=f(x_2)$.

So, we have $\frac{x_1-2}{x_1-3}=\frac{x_2-2}{x_2-3}$

$\Rightarrow(x_1-2)(x_2-3)=(x_2-2)(x_1-3)\quad\Rightarrow x_1=x_2$.

That implies, f(x) is one-one.

Let $y=f(x)=\frac{x-2}{x-3}$ for $y\in B$.

$\Rightarrow xy-3y=x-2$

$\Rightarrow 2-3y=x-xy$

$\Rightarrow x=\frac{2-3y}{1-y};\ y\neq 1$.

Since for all $y\in B$, $x\in A$ i.e., Range of f = Codomain of f.

Hence f(x) is onto.

As f is one-one and onto both so, it is bijective.

22. If $A=\begin{bmatrix}2&3\\-1&2\end{bmatrix}$, then show that $A^2-4A+7I=O$.

Sol. $A^2=\begin{bmatrix}2&3\\-1&2\end{bmatrix}\begin{bmatrix}2&3\\-1&2\end{bmatrix}=\begin{bmatrix}1&12\\-4&1\end{bmatrix}$, $4A=4\begin{bmatrix}2&3\\-1&2\end{bmatrix}=\begin{bmatrix}8&12\\-4&8\end{bmatrix}$, $7I=7\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}7&0\\0&7\end{bmatrix}$

LHS : $A^2-4A+7I=\begin{bmatrix}1&12\\-4&1\end{bmatrix}-\begin{bmatrix}8&12\\-4&8\end{bmatrix}+\begin{bmatrix}7&0\\0&7\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}=O=RHS$.

Remark Here it is given that $A^2-4A+7I=0$, however it should be $A^2-4A+7I=O$.

23. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2-5xy+y^3=76$, then find $\frac{dy}{dx}$.

Sol. (a) Let $y=\left(\frac{5^x}{x^5}\right)$

On differentiating w.r.t. x, we get $\frac{dy}{dx}=\frac{x^5\frac{d(5^x)}{dx}-5^x\frac{d(x^5)}{dx}}{(x^5)^2}$

$\Rightarrow \frac{dy}{dx}=\frac{x^5\times 5^x\log 5-5\times 5^x\times x^4}{x^{10}}$

$\therefore \frac{dy}{dx}=\frac{(x\log 5-5)\times 5^x}{x^6}$.

OR

(b) Given $-2x^2-5xy+y^3=76$

On differentiating w.r.t. x, we get $-4x-5\left(x\frac{dy}{dx}+y\times 1\right)+3y^2\frac{dy}{dx}=0$

$\Rightarrow (-5x+3y^2)\frac{dy}{dx}=4x+5y$

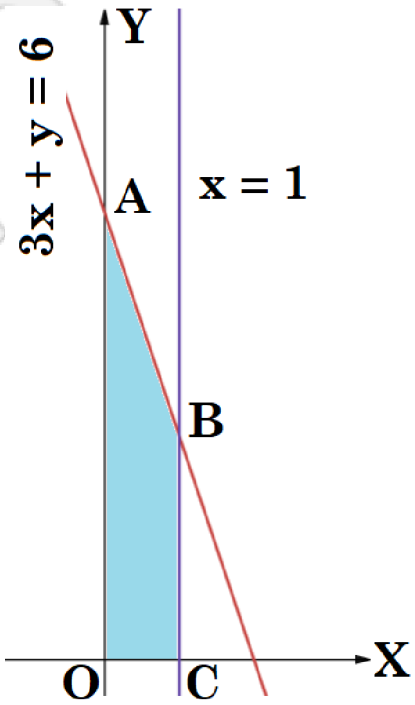
$\therefore \frac{dy}{dx}=\frac{4x+5y}{3y^2-5x}$.

24. In a Linear Programming Problem, the objective function $Z=5x+4y$ needs to be maximized under constraints $3x+y\le 6$, $x\le 1$; $x, y\ge 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.

Sol. Consider the graph shown below.

The corner points are O(0, 0), A(0, 6), B(1, 3) and C(1, 0).

Note : We do *not* need to find the maximum value of Z for this LPP.



25. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?

Sol. (a) Total number of blocks : 10.

As X denotes the number written in the block so, values of X are 0, 1, 2, 3.

Now $P(X=0)=\frac{2}{10}$, $P(X=1)=\frac{3}{10}$, $P(X=2)=\frac{4}{10}$ and $P(X=3)=\frac{1}{10}$.

Table for probability distribution is given below.

X	0	1	2	3
P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

$$\text{Mean}=\sum X\,P(X)=0\times\frac{2}{10}+1\times\frac{3}{10}+2\times\frac{4}{10}+3\times\frac{1}{10}=\frac{14}{10}=1.4.$$

OR

(b) Total people : 8000.

Let A be the event that a randomly chosen individual is a woman.

Also let B be the event that a randomly chosen individual works outside the village.

Here $P(A)=\frac{4000}{8000}=\frac{4}{8}$, $P(B)=\frac{3000}{8000}=\frac{3}{8}$, $P(A\cap B)=\frac{1200}{8000}=\frac{12}{80}$

[No. of women working outside the village $=4000\times30\%=1200$

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{4}{8}+\frac{3}{8}-\frac{12}{80}=\frac{58}{80}=\frac{29}{40}$.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. (a) Show that the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=4x^3-5$, $\forall\, x\in\mathbb{R}$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that $R=\{(x,y):xy$ is a square of a natural number; $x, y\in N\}$. Determine if the relation R is an equivalence relation.

Sol. (a) Let $x_1, x_2\in\mathbb{R}$ such that $f(x_1)=f(x_2)$.

Then $4x_1^3-5=4x_2^3-5$

$\Rightarrow x_1^3=x_2^3$

$\Rightarrow x_1^3-x_2^3=0$

$\Rightarrow (x_1-x_2)(x_1^2+x_1x_2+x_2^2)=0$

So $(x_1-x_2)=0$

$\Rightarrow x_1=x_2$

Note that $(x_1^2+x_1x_2+x_2^2)=0$ only when both x_1 and x_2 are 0 i.e., $x_1=x_2=0$.

We can conclude that, $x_1=x_2$.

So, $f(x)$ is one-one.

Further let $y=f(x)$, $y\in\mathbb{R}$.

Then $y=4x^3-5$

$\Rightarrow x=\left(\frac{y+5}{4}\right)^{\frac{1}{3}}\in\mathbb{R}\ \forall\ y\in\mathbb{R}$. That is, range = codomain.

Hence, $f(x)$ is onto.

Alternatively, for one-one : $f'(x) = 12x^2 \geq 0 \ \forall x \in \mathbb{R}$. That is, $f(x)$ is an increasing function. Since an increasing function never takes the same value twice, so the function f is one-one.

OR

(b) Since $x.x = x^2 \ \forall x \in N$. So, clearly $(x, x) \in R \ \forall x \in N$. Therefore, R is reflexive.

If xy is a square of a natural number, then yx is also a square of the same natural number.

That is, $(x, y) \in R$ implies $(y, x) \in R \ \forall x, y \in N$. Therefore, R is symmetric.

Now if $(x, y) \in R$ and $(y, z) \in R \ \forall x, y, z \in N$, then it means xy is a square of a natural number and yz is a square of a natural number.

Let $xy = a^2$, $yz = b^2$. Then, $xz = \dfrac{a^2b^2}{y^2}$.

For xz to be a perfect square, y^2 must perfectly divide a^2b^2, which is always true.

It clearly means, xz is also a square of a natural number. That is, $(x, z) \in R$. Therefore, R is transitive.

Hence, R is an equivalence relation as it is reflexive, symmetric and transitive.

27. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹150 (Chemistry), ₹175 (Physics) and ₹180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹35, 000, what profit did he earn after the sale of two days?

Sol. (a) Given $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$.

On rewriting, we get $2x + 5y = 1$ and $3x + 2y = 7$.

Let $A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 7 \end{bmatrix}$. Note that, $AX = B \ \Rightarrow X = A^{-1}B$ ...(i)

Now $|A| = 2 \times 2 - 3 \times 5 = -11$ and $\text{adj.}A = \begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix}$. So, $A^{-1} = \dfrac{\text{adj.}A}{|A|} = \dfrac{1}{11}\begin{bmatrix} -2 & 5 \\ 3 & -2 \end{bmatrix}$.

By (i), $X = \dfrac{1}{11}\begin{bmatrix} -2 & 5 \\ 3 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 7 \end{bmatrix} = \dfrac{1}{11}\begin{bmatrix} -2+35 \\ 3-14 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

By equality of matrices, we get $x = 3$, $y = -1$.

Hence, the common point to the paths of the ants is (3, –1).

OR

(b) Let $A = \begin{bmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{bmatrix}$, $B = \begin{bmatrix} 150 \\ 175 \\ 180 \end{bmatrix}$

Consider $AB = \begin{bmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{bmatrix}\begin{bmatrix} 150 \\ 175 \\ 180 \end{bmatrix} = \begin{bmatrix} 7500+10500+6300 \\ 6000+7875+9000 \end{bmatrix} = \begin{bmatrix} 24300 \\ 22875 \end{bmatrix}$.

Therefore, total sale price = Day I + Day II = 24300 + 22875 = 47175 (in ₹).

Hence, the total profit = Sale price − Cost price = $47175 - 35000 = 12175$ (in ₹).

28. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}$ with respect to x.

Sol. $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}$

On squaring both the sides, we get $y^2 = \log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}$

On differentiating w.r.t. x, we get $2y\times\frac{dy}{dx} = \frac{d}{dx}\left[\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}\right]$

$$\Rightarrow 2y\times\frac{dy}{dx} = \frac{1}{\sin\left(\frac{x^3}{3}-1\right)}\times\frac{d}{dx}\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}$$

$$\Rightarrow 2y\times\frac{dy}{dx} = \frac{1}{\sin\left(\frac{x^3}{3}-1\right)}\times\cos\left(\frac{x^3}{3}-1\right)\times\frac{d}{dx}\left(\frac{x^3}{3}-1\right)$$

$$\Rightarrow 2y\times\frac{dy}{dx} = \cot\left(\frac{x^3}{3}-1\right)\times\left(\frac{3x^2}{3}-0\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2}{2y}\times\cot\left(\frac{x^3}{3}-1\right) \text{ or, } \frac{dy}{dx} = \frac{x^2}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}\times\cot\left(\frac{x^3}{3}-1\right).$$

29. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

Sol. Let one of the two positive integers be x so, the other one must be $\frac{289}{x}$.

Now let $S = x + \frac{289}{x}$.

Then $\frac{dS}{dx} = 1 - \frac{289}{x^2}$ and $\frac{d^2S}{dx^2} = \frac{2\times289}{x^3}$.

For $\frac{dS}{dx} = 0$, $1 - \frac{289}{x^2} = 0$ $\Rightarrow x^2 - 289 = 0$ $\Rightarrow x^2 = 289$

$\therefore x = 17 \in Z^+$

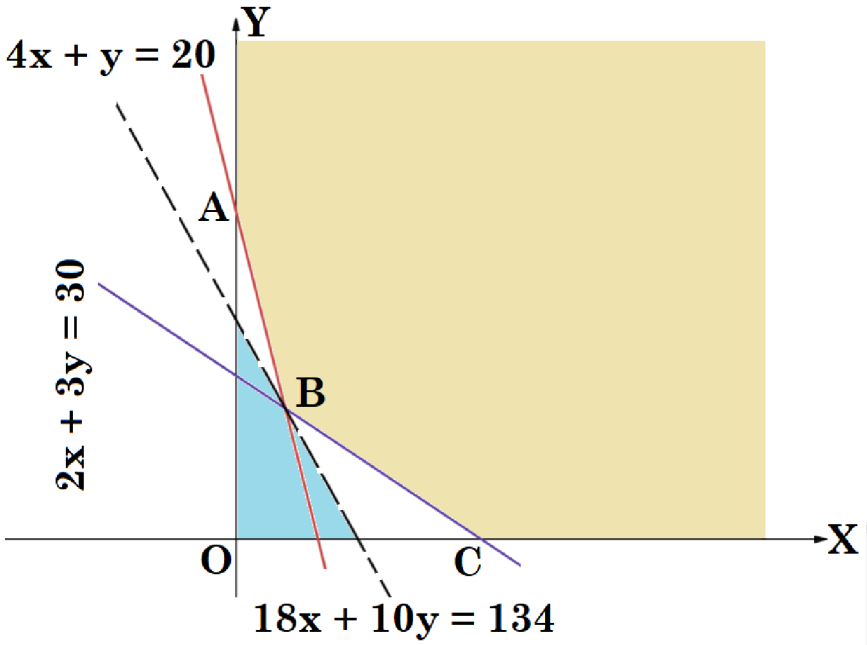
Note that $\left(\frac{d^2S}{dx^2}\right)_{\text{at } x=17} = \frac{2\times289}{17^3} = \frac{2}{17} > 0$.

So, S is least when $x = 17$.

Hence, both the numbers are $x = 17$ and $\frac{289}{x} = \frac{289}{17} = 17$.

30. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints $4x + y \geq 20$, $2x + 3y \geq 30$; $x, y \geq 0$, find the minimum value of Z.

Sol. Consider the graph shown below.



The corner points are $A(0, 20)$, $B(3, 8)$ and $C(15, 0)$.

Corner Points	Value of Z
A(0, 20)	200
B(3, 8)	134 ← Minimum
C(15, 0)	270

Since the feasible region is unbounded so, $Z = 134$ may or may not be the minimum value.
To check, let $18x + 10y < 134$.
Note that there is no common point between the feasible region and $18x + 10y < 134$. So, $Z = 134$ is the minimum value and it is obtained at (3, 8).

31. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

OR

(b) Find the shortest distance between the lines: $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$;
$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k})$.

Sol. (a) Here $\vec{b} + \vec{c} = (2 + \lambda)\hat{i} - 6\hat{j} + 2\hat{k}$.

According to the question, $(\hat{i} - \hat{j} + 2\hat{k}).\dfrac{(2+\lambda)\hat{i} - 6\hat{j} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + (-6)^2 + 2^2}} = 1$

$\Rightarrow (2 + \lambda) + 6 + 4 = \sqrt{(2+\lambda)^2 + 40}$

On squaring both the sides, we get

$(12 + \lambda)^2 = (2 + \lambda)^2 + 40$

$\Rightarrow 144 + 24\lambda + \lambda^2 = 4 + 4\lambda + \lambda^2 + 40$

$\Rightarrow 20\lambda = -100$

$\therefore \lambda = -5$.

OR

(b) Here $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$; $\vec{a}_2 = \hat{i} + 4\hat{k}$, $\vec{b}_2 = 3\hat{i} - 6\hat{j} + 9\hat{k}$.

Now $\vec{a}_2 - \vec{a}_1 = -\hat{i} + \hat{j} + \hat{k}$; $\vec{b}_1 \| \vec{b}_2$ (as $\vec{b}_2 = 3\vec{b}_1$). Take $\vec{b}_1 = \vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$.

Also $(\vec{a}_2-\vec{a}_1)\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\-1&1&1\\1&-2&3\end{vmatrix}=5\hat{i}+4\hat{j}+\hat{k}$.

$\therefore$ Shortest distance, $S.D.=\dfrac{|(\vec{a}_2-\vec{a}_1)\times\vec{b}|}{|\vec{b}|}=\dfrac{|5\hat{i}+4\hat{j}+\hat{k}|}{|\hat{i}-2\hat{j}+3\hat{k}|}=\dfrac{\sqrt{25+16+1}}{\sqrt{1+4+9}}=\sqrt{\dfrac{42}{14}}=\sqrt{3}$ units.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. (a) Find $\int\dfrac{x^2+1}{(x^2+2)(2x^2+1)}dx$.

OR

(b) Evaluate $\int_0^{\pi}\dfrac{x\tan x}{\sec x+\tan x}dx$.

Sol. (a) $\int\dfrac{x^2+1}{(x^2+2)(2x^2+1)}dx=\int\dfrac{(x^2+2)-1}{(x^2+2)(2x^2+1)}dx$

$$=\int\left[\frac{(x^2+2)}{(x^2+2)(2x^2+1)}-\frac{1}{(x^2+2)(2x^2+1)}\right]dx$$

$$=\int\frac{1}{(2x^2+1)}dx-\int\frac{1}{3}\left[\frac{2}{2x^2+1}-\frac{1}{x^2+2}\right]dx$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}x)-\frac{1}{3}\left[2\times\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}x)-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right]+C$$

$$=\left(1-\frac{2}{3}\right)\times\frac{1}{\sqrt{2}}\tan^{-1}(\sqrt{2}x)+\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C$$

$$=\frac{1}{3\sqrt{2}}\tan^{-1}(\sqrt{2}x)+\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)+C.$$

OR

(b) Let $I=\int_0^{\pi}\dfrac{x\tan x}{\sec x+\tan x}dx$

$$\Rightarrow I=\int_0^{\pi}\frac{x\dfrac{\sin x}{\cos x}}{\dfrac{1}{\cos x}+\dfrac{\sin x}{\cos x}}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{x\sin x}{1+\sin x}dx\quad\ldots(i)$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+\sin(\pi-x)}dx$$

$$\Rightarrow I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\sin x}dx\quad\ldots(ii)$$

Adding (i) and (ii), we get : $2I=\int_0^{\pi}\dfrac{x\sin x}{1+\sin x}dx+\int_0^{\pi}\dfrac{(\pi-x)\sin x}{1+\sin x}dx$

$$\Rightarrow I=\frac{\pi}{2}\int_0^{\pi}\frac{\sin x}{1+\sin x}dx$$

Consider $f(x)=\frac{\sin x}{1+\sin x}\Rightarrow f(\pi-x)=\frac{\sin(\pi-x)}{1+\sin(\pi-x)}=\frac{\sin x}{1+\sin x}=f(x)$.

$$\therefore I=\frac{\pi}{2}\times 2\int_0^{\pi/2}\frac{\sin x}{1+\sin x}dx$$

$$\Rightarrow I=\pi\int_0^{\pi/2}\left(1-\frac{1}{1+\sin x}\right)dx$$

$$\Rightarrow I=\pi\int_0^{\pi/2}\left(1-\frac{1}{1+\cos(\pi/2-x)}\right)dx$$

$$\Rightarrow I=\pi\int_0^{\pi/2}\left(1-\frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)dx$$

$$\Rightarrow I=\pi\left[x+\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right]_0^{\pi/2}$$

$$\Rightarrow I=\pi\left[\frac{\pi}{2}+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right)\right]-\pi\left[0+\tan\left(\frac{\pi}{4}-\frac{0}{2}\right)\right]$$

$$\Rightarrow I=\pi\left[\frac{\pi}{2}+0\right]-\pi\left[0+\tan\left(\frac{\pi}{4}\right)\right]$$

$$\Rightarrow I=\pi\left[\frac{\pi}{2}\right]-\pi$$

$$\therefore I=\pi\left(\frac{\pi}{2}-1\right).$$

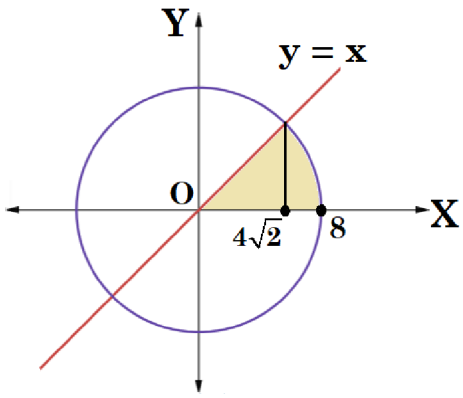
33. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

Sol. Since table is circular and its radius is 8 cm.

Equation of circle $x^2+y^2=8^2$, also as the scratch is passing through (0, 0) and making an angle of $\frac{\pi}{4}$ so, its equation will be $y=x$.

When we solve the curves, we get

$2x^2=64 \Rightarrow x=\pm 4\sqrt{2}$.



$$\text{Required area}=\int_0^{4\sqrt{2}}x\,dx+\int_{4\sqrt{2}}^{8}\sqrt{64-x^2}\,dx$$

$$=\frac{1}{2}\left[x^2\right]_0^{4\sqrt{2}}+\left[\frac{x}{2}\sqrt{64-x^2}+\frac{64}{2}\sin^{-1}\frac{x}{8}\right]_{4\sqrt{2}}^{8}$$

$$= \frac{1}{2}[32-0] + \left[\left(0+32\times\frac{\pi}{2}\right) - \left(\frac{4\sqrt{2}}{2}\times 4\sqrt{2} + 32\times\frac{\pi}{4}\right)\right]$$

$$= 16 + 16\pi - 8\pi - 16 = 8\pi \text{ Sq.cm}.$$

34. Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

Sol. Rewriting the D.E., we get $\frac{dy}{dx} + 2y = \cos x$

On comparing with $\frac{dy}{dx} + P(x)\,y = Q(x)$, we get $P(x) = 2$, $Q(x) = \cos x$

Integration factor $= e^{\int 2dx} = e^{2x}$.

So, the solution is given by $y e^{2x} = \int \cos x\, e^{2x} dx + C$ ...(i)

Consider $I = \int \cos x\, e^{2x} dx$ [Apply By Parts method

$$\Rightarrow I = \cos x \int e^{2x} dx - \int \left(\frac{d}{dx}[\cos x] \int e^{2x} dx\right) dx$$

$$\Rightarrow I = \frac{e^{2x}}{2}\cos x + \frac{1}{2}\int \sin x\, e^{2x} dx$$ [Again apply By Parts method

$$\Rightarrow I = \frac{e^{2x}}{2}\cos x + \frac{1}{2}\left[\sin x \int e^{2x} dx - \int \left(\frac{d}{dx}\{\sin x\} \int e^{2x} dx\right) dx\right]$$

$$\Rightarrow I = \frac{e^{2x}}{2}\cos x + \frac{1}{2}\left[\frac{e^{2x}}{2}\sin x - \frac{1}{2}\int \cos x\, e^{2x} dx\right]$$

$$\Rightarrow I = \frac{e^{2x}}{2}\cos x + \frac{e^{2x}}{4}\sin x - \frac{1}{4}I$$

$$\Rightarrow \frac{5}{4}I = \frac{e^{2x}}{4}[2\cos x + \sin x] \qquad \Rightarrow I = \frac{e^{2x}}{5}[2\cos x + \sin x]$$

By (i), the required solution is given by : $y e^{2x} = \frac{e^{2x}}{5}[2\cos x + \sin x] + C$.

That is, $y = \frac{2\cos x + \sin x}{5} + C e^{-2x}$.

35. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

OR

(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

Sol. (a) Given point is P(1, 2, 3).

Given line is $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ i.e., $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2} = \lambda$.

The coordinates of any random point on the line is $Q(3\lambda-2,\ 2\lambda-1,\ 2\lambda+3)$.

According to question, $PQ = 3\sqrt{2} = \sqrt{(3\lambda-3)^2 + (2\lambda-3)^2 + (2\lambda)^2}$

$\Rightarrow 9\lambda^2 - 18\lambda + 9 + 4\lambda^2 - 12\lambda + 9 + 4\lambda^2 = 18 \quad \Rightarrow 17\lambda^2 - 30\lambda = 0$

$\Rightarrow \lambda(17\lambda - 30) = 0$

$\Rightarrow \lambda = 0$ or $\lambda = \frac{30}{17}$

We shall now replace these values of λ in $Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$.

Hence, the required point is $(-2, -1, 3)$ or $\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$.

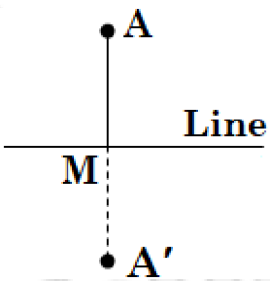
OR

(b) Given line is $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$ i.e., $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3} = \lambda$. Its d.r.'s are 1, 2, –3.

Let $A(-1, 5, 2)$. Let M be foot of perpendicular on the given line.

Any arbitrary point on the line is $M(\lambda + 2, 2\lambda, -3\lambda + 2)$.

Direction ratios of AM : $\lambda + 3, 2\lambda - 5, -3\lambda$.



Since AM is perpendicular to the given line.

So by using $a_1a_2 + b_1b_2 + c_1c_2 = 0$ for $\perp$ lines, we get

$1(\lambda + 3) + 2(2\lambda - 5) + (-3)(-3\lambda) = 0$

$\Rightarrow 14\lambda = 7 \quad \therefore \lambda = \frac{1}{2}$

So, $M\left(\frac{5}{2}, 1, \frac{1}{2}\right)$ is the **foot of perpendicular** of the point A in line.

Let image of point A in the line be $A'(h, p, s)$.

Since M is the mid-point of AA′, so $M\left(\frac{h-1}{2}, \frac{p+5}{2}, \frac{s+2}{2}\right) = M\left(\frac{5}{2}, 1, \frac{1}{2}\right)$.

On comparing the respective coefficients, we obtain : $h = 6, p = -3, s = -1$.

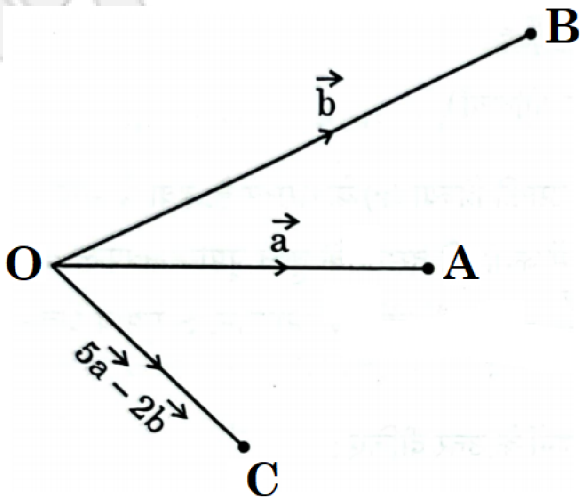
Hence, $A'(6, -3, -1)$ is the **image** of point A in the line.

Also, the required length $(AA') = \sqrt{(6+1)^2 + (-3-5)^2 + (-1-2)^2} = \sqrt{122}$ units.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his pre-decided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions.

(i) Complete the given figure to explain their entire movement plan along the respective vectors.

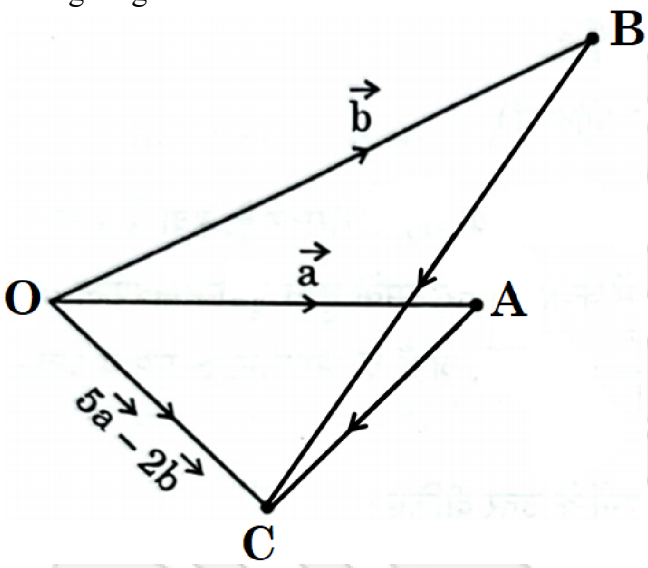
(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

(iii) (a) If $\vec{a}.\vec{b}=1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a}\times\vec{b}|$.

OR

(iii) (b) If $\vec{a}=2\hat{i}-\hat{j}+4\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$, then find a unit vector perpendicular to $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})$.

Sol. **(i)** Consider the following diagram.



(ii) $\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=5\vec{a}-2\vec{b}-\vec{a}=4\vec{a}-2\vec{b}$ and $\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=5\vec{a}-2\vec{b}-\vec{b}=5\vec{a}-3\vec{b}$.

(iii) (a) Given that $|\overrightarrow{OA}|=|\vec{a}|=1$, $|\overrightarrow{OB}|=|\vec{b}|=2$.

$$\text{Required angle}=\cos^{-1}\frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}=\cos^{-1}\frac{1}{1\times 2}=\cos^{-1}\frac{1}{2}=\frac{\pi}{3}.$$

$$\text{Also } |\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\frac{\pi}{3}=1\times 2\times\frac{\sqrt{3}}{2}=\sqrt{3}.$$

OR

(iii) (b) $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\vec{a}\times\vec{a}-\vec{a}\times\vec{b}+\vec{b}\times\vec{a}-\vec{b}\times\vec{b}=\vec{0}+\vec{b}\times\vec{a}+\vec{b}\times\vec{a}+\vec{0}=2\vec{b}\times\vec{a}$

$$\text{So, } \vec{b}\times\vec{a}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&-1\\2&-1&4\end{vmatrix}=3\hat{i}-2\hat{j}-2\hat{k}.$$

Now a unit vector perpendicular to $(\vec{a}+\vec{b})$ and $(\vec{a}-\vec{b})=\pm\dfrac{(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})}{|(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})|}=\pm\dfrac{2\vec{b}\times\vec{a}}{|2\vec{b}\times\vec{a}|}$

$$=\pm\frac{2\vec{b}\times\vec{a}}{2|\vec{b}\times\vec{a}|}=\pm\frac{(3\hat{i}-2\hat{j}-2\hat{k})}{\sqrt{9+4+4}}$$

$$=\pm\frac{(3\hat{i}-2\hat{j}-2\hat{k})}{\sqrt{17}}.$$

37. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions.

(i) Write the order and degree of the given differential equation.

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm.

(iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

OR

(iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

Sol. **(i)** For D.E. $\frac{dV}{dt} = kS$, the order is 1 and degree is 1.

(ii) We have $\frac{dr}{dt} = \frac{2}{3}k$

$\Rightarrow dr = \frac{2}{3}k\,dt$

On integrating, $\int dr = \int \frac{2}{3}k\,dt$

$\therefore r = \frac{2k}{3} \times t + C$.

It is given that $r(0) = 5$ mm so, $5 = \frac{2k}{3} \times 0 + C \qquad \Rightarrow C = 5$

Hence, the required solution is $r = \frac{2k}{3} \times t + 5$.

(iii) (a) Given that $r = 3$ mm when $t = 1$ hour. So, $3 = \frac{2k}{3} \times 1 + 5$

$\Rightarrow -2 = \frac{2k}{3} \qquad \therefore k = -3$.

Now for $r = 0$ mm, we get $0 = \frac{2(-3)}{3} \times t + 5 \quad \Rightarrow -5 = -2 \times t$

$\therefore t = \frac{5}{2}$ hours.

OR

(iii) (b) Given that $r = 1$ mm when $t = 1$ hour. So, $1 = \frac{2k}{3} \times 1 + 5$

$\Rightarrow -4 = \frac{2k}{3} \quad \therefore k = -6$.

Now for $r = 0$ mm, we get $0 = \frac{2(-6)}{3} \times t + 5 \quad \Rightarrow -5 = -4 \times t$

$\therefore t = \frac{5}{4}$ hours.

38. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50% respectively.

Based on upon the above information, answer the following questions:

(i) A person was tested randomly. What is the probability that he/she has contracted the disease?

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2?

Sol. Let D be the event of people contracting the disease.

Now $P(A_1) = \frac{700}{1000} = \frac{7}{10}$, $P(A_2) = \frac{200}{1000} = \frac{2}{10}$, $P(A_3) = \frac{100}{1000} = \frac{1}{10}$;

$P(D \mid A_1) = 25\%$, $P(D \mid A_2) = 35\%$, $P(D \mid A_3) = 50\%$.

(i) By using law of total probability, we get

$P(D) = P(A_1) \times P(D \mid A_1) + P(A_2) \times P(D \mid A_2) + P(A_3) \times P(D \mid A_3)$

$\Rightarrow P(D) = \frac{7}{10} \times \frac{25}{100} + \frac{2}{10} \times \frac{35}{100} + \frac{1}{10} \times \frac{50}{100}$

$\Rightarrow P(D) = \frac{175 + 70 + 50}{1000} = \frac{295}{1000} = \frac{59}{200}$ or 29.5%.

(ii) Using Bayes' theorem, $P(A_2 \mid \bar{D}) = \frac{P(\bar{D} \mid A_2) P(A_2)}{P(\bar{D})}$

$\Rightarrow P(A_2 \mid \bar{D}) = \frac{[1 - P(D \mid A_2)] P(A_2)}{1 - 0.295}$

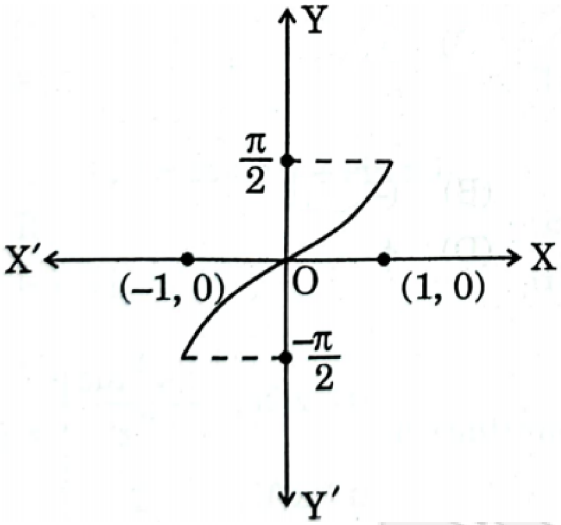
$\Rightarrow P(A_2 \mid \bar{D}) = \frac{[1 - 0.35] \times 0.2}{0.705}$

$\Rightarrow P(A_2 \mid \bar{D}) = \frac{0.65 \times 0.2}{0.705} \quad \Rightarrow P(A_2 \mid \bar{D}) = \frac{0.13}{0.705}$

$\therefore P(A_2 \mid \bar{D}) = \frac{130}{705} = \frac{26}{141}$.

☑ Q.P. Code - 65/7/2

01. Study the given graph. It illustrates



(a) $y = \tan^{-1} x$ (b) $y = \cos^{-1} x$ (c) $y = \sec^{-1} x$ (d) $y = \sin^{-1} x$

Sol. (d) The graph illustrates $y = \sin^{-1} x$.

08. If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2}, & \text{if } x \neq 0 \\ 1, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of 'a' is

(a) ± 1 (b) -1 (c) 0 (d) 1

Sol. (a) As f is continuous at $x = 0$ so, $\lim\limits_{x \to 0} f(x) = f(0)$

That is, $\lim\limits_{x \to 0} \dfrac{\sin^2 ax}{x^2} = 1 \quad \Rightarrow \lim\limits_{ax \to 0} \dfrac{\sin^2 ax}{(ax)^2} \times a^2 = 1$

$\Rightarrow 1^2 \times a^2 = 1$

$\therefore a = \pm 1$

10. If R be a relation defined as $a\,R\,b$ iff $|a - b| > 0$; $a, b \in \mathbb{R}$, then R is

(a) reflexive (b) symmetric (c) transitive (d) symmetric and transitive

Sol. (b) Note that, $a\,R\,a$ is not possible, so R is not reflexive.

Also $a\,R\,b$ implies $b\,R\,a$. So, R is symmetric.

Let $a = 2, b = 4, c = 2$. Then $2\,R\,4$ and $4\,R\,2$ is true. But $2\,R\,2$ is not possible.

So, R is not transitive.

14. $\int \dfrac{a^x}{\sqrt{1 - a^{2x}}} dx$ is equal to

(a) $\dfrac{\sin^{-1}(a^x)}{\log_e a} + C$ (b) $\log_e (1 - a^{2x}) + C$ (c) $\dfrac{\cos^{-1}(a^x)}{\log_e a} + C$ (d) $\dfrac{\sin^{-1}(a^x)}{a^x} + C$

Sol. (a) Put $a^x = t \Rightarrow a^x dx = \dfrac{dt}{\log_e a}$.

So, $\int \dfrac{a^x}{\sqrt{1 - a^{2x}}} dx = \dfrac{1}{\log_e a} \int \dfrac{1}{\sqrt{1 - t^2}} dt = \dfrac{1}{\log_e a} \sin^{-1} t + C = \dfrac{\sin^{-1}(a^x)}{\log_e a} + C$.

17. If $P(A) = \dfrac{1}{5}$, $P(B) = \dfrac{3}{5}$ and $P(A \mid B) = \dfrac{2}{5}$, then $P(A' \cap B')$ is

(a) $\frac{11}{25}$ (b) $\frac{19}{25}$ (c) $\frac{8}{25}$ (d) $\frac{6}{25}$

Sol. (a) $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - [P(A) + P(B) - P(A \cap B)]$

$\Rightarrow P(A' \cap B') = 1 - [P(A) + P(B) - P(A \mid B) \times P(B)]$

$\Rightarrow P(A' \cap B') = 1 - \left[\frac{1}{5} + \frac{3}{5} - \frac{2}{5} \times \frac{3}{5}\right]$

$\therefore P(A' \cap B') = 1 - \frac{14}{25} = \frac{11}{25}$.

Remark Here $P(A \cap B) = \frac{6}{25}$, $P(A) = \frac{1}{5}$ i.e., $P(A \cap B) > P(A)$, which is **not** possible.

22. If $\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 2 \\ -5 \\ -17 \end{bmatrix}$, then find matrix A.

Sol. Let $A = \begin{bmatrix} x \\ y \end{bmatrix}$.

Now $\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 2 \\ -5 \\ -17 \end{bmatrix}$ implies, $\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -17 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 3x - y \\ 0x + y \\ 2x - 3y \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ -17 \end{bmatrix}$

By equality of matrices, we get $3x - y = 2$, $y = -5$, $2x - 3y = -17$

Clearly, no values of x and y simultaneously satisfy all the equations obtained.

Hence, matrix A can not be obtained.

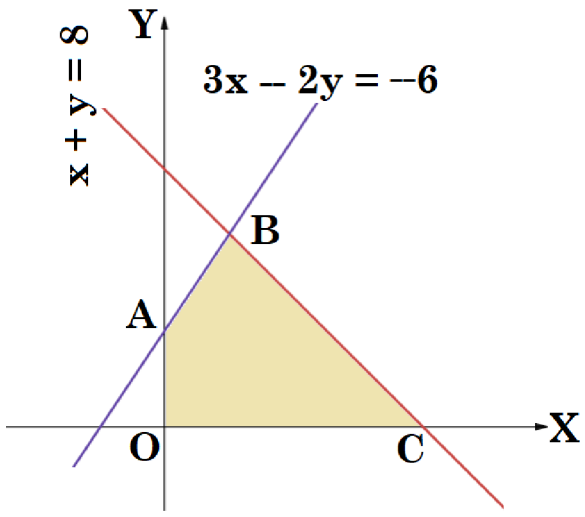
Remark The given data in this matrix equation is incorrect.

24. In a Linear Programming Problem (LPP) for objective function $Z = 14x - 10y$ subject to constraints $x + y \le 8$, $3x - 2y \ge -6$; $x, y \ge 0$ shade the feasible region and mark the corner points in a neatly drawn graph.

Sol. Consider the graph shown below.

The corner points are O (0, 0), A (0, 3), B (2, 6) and C (8, 0).

Note : We do *not* need to find the maximum / minimum value of Z for this LPP.



28. Show that the derivative of $\tan^{-1}(\sec x + \tan x), \left[-\frac{\pi}{2} < x < \frac{\pi}{2}\right]$ with respect to x is equal to $\frac{1}{2}$.

Sol. Let $y = \tan^{-1}(\sec x + \tan x)$

$$\Rightarrow y = \tan^{-1}\left(\frac{1}{\cos x} + \frac{\sin x}{\cos x}\right) \qquad \Rightarrow y = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$$

$$\Rightarrow y = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right] = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right]$$

$$\Rightarrow y = \tan^{-1}\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right] = \tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right] = \tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$\Rightarrow y = \left(\frac{\pi}{4}+\frac{x}{2}\right) \qquad \left[\because \tan^{-1}\tan x = x,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)\right.$$

$$\therefore \frac{dy}{dx} = 0 + \frac{1}{2} = \frac{1}{2}.$$

Note that, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ so, $-\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{4}$.

It further implies, $0 < \frac{\pi}{4} + \frac{x}{2} < \frac{\pi}{2}$ i.e., $\left[\frac{\pi}{4}+\frac{x}{2}\right] \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

29. Find dimensions of a rectangle of perimeter 12 cm which will generate maximum volume when swept along a circular rotation keeping the shorter side fixed as the axis.

Sol. Let x and y represent the length of rectangle and its breadth respectively.

So, $2(x+y) = 12 \quad \Rightarrow x + y = 6 \ \ldots(i)$

If rectangle is rotated by keeping shorter side (y) fixed as the axis, then it forms a cylinder in space with base radius of the cylinder as x and height as y.

Now volume of cylinder, $V = \pi x^2 y = \pi x^2(6-x) = \pi(6x^2 - x^3)$ [By (i)

$$\Rightarrow \frac{dV}{dx} = \pi(12x - 3x^2) \text{ and } \frac{d^2V}{dx^2} = \pi(12 - 6x)$$

For critical points, $\frac{dV}{dx} = 0$, $\pi(12x - 3x^2) = 0 \quad \Rightarrow 3x(4-x) = 0$

As $x \neq 0$ so, $x = 4$ cm.

Note that, $\left(\frac{d^2V}{dx^2}\right)_{\text{at } x=4 \text{ cm}} = \pi(12 - 6\times 4) = -12\pi < 0$.

So, V is maximum at $x = 4$ cm.

Hence, the length of rectangle is $x = 4$ cm and breadth will be $y = 6 - x = 2$ cm.

32. (a) Find $\int \frac{x}{(x-1)(x^2+4)}dx$.

OR

(b) Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}dx$.

Sol. (a) Consider $\frac{x}{(x-1)(x^2+4)} = \frac{A}{(x-1)} + \frac{B(2x)+C}{(x^2+4)}$

$\Rightarrow x = A(x^2+4) + B(2x)(x-1) + C(x-1)$

On comparing the coefficients of like terms from both the sides, we get $A = \frac{1}{5}$, $B = -\frac{1}{10}$, $C = \frac{4}{5}$.

Now $\int \frac{x}{(x-1)(x^2+4)}dx = \int \left[\frac{1}{5}\times\frac{1}{x-1} - \frac{1}{10}\times\frac{2x}{(x^2+4)} + \frac{4}{5}\times\frac{1}{(x^2+4)}\right]dx$

$= \frac{1}{5}\times\log|x-1| - \frac{1}{10}\times\log|x^2+4| + \frac{4}{5}\times\frac{1}{2}\tan^{-1}\frac{x}{2} + C$

$= \frac{1}{5}\log|x-1| - \frac{1}{10}\log|x^2+4| + \frac{2}{5}\tan^{-1}\frac{x}{2} + C.$

OR

(b) Let $I = \int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$ ...(i)

$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}dx$

$\Rightarrow I = \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}dx$ ...(ii)

Adding (i) & (ii), we get $2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

Let $f(x) = \frac{\sin x}{1+\cos^2 x}$. As $f(\pi-x) = \frac{\sin(\pi-x)}{1+\cos^2(\pi-x)} = \frac{\sin x}{1+\cos^2 x} = f(x)$.

So, by using $\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$, $f(2a-x) = f(x)$, we have $2I = \pi\times 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$

$\Rightarrow I = \pi\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$

Put $\cos x = t \Rightarrow \sin x\,dx = -dt$. When $x = 0 \Rightarrow t = 1$, when $x = \frac{\pi}{2} \Rightarrow t = 0$.

So, $I = \pi\int_1^0 \frac{-1}{1+t^2}dt = \pi\int_0^1 \frac{1}{1+t^2}dt = \pi\left[\tan^{-1}t\right]_0^1 = \pi\left[\frac{\pi}{4}-0\right]$

$\therefore I = \frac{\pi^2}{4} = \left(\frac{\pi}{2}\right)^2$.

34. Solve the differential equation $(x^2+y^2)dx + xy\,dy = 0$, $y(1) = 1$.

Sol. Rewriting the D.E., we get $\frac{dy}{dx} = -\frac{(x^2+y^2)}{xy}$ i.e., $\frac{dy}{dx} = -\frac{x}{y} - \frac{y}{x}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

So, $v + x\frac{dv}{dx} = -\frac{1}{v} - v$

$$\Rightarrow x\frac{dv}{dx}=-\frac{1}{v}-2v=-\left(\frac{1+2v^2}{v}\right)$$

$$\Rightarrow \int\left(\frac{v}{1+2v^2}\right)dv=-\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{4}\int\left(\frac{4v}{1+2v^2}\right)dv=-\int\frac{dx}{x}$$

$$\Rightarrow \frac{1}{4}\log\left|1+2v^2\right|=-\log|x|+\log k,\ k>0$$

$$\Rightarrow \frac{1}{4}\log\left|\frac{x^2+2y^2}{x^2}\right|=\log\left|\frac{k}{x}\right| \quad \Rightarrow \log\left|\frac{x^2+2y^2}{x^2}\right|=4\log\left|\frac{k}{x}\right|$$

$$\Rightarrow \log\left|\frac{x^2+2y^2}{x^2}\right|=\log\left|\frac{k^4}{x^4}\right| \quad \Rightarrow \frac{x^2+2y^2}{x^2}=\frac{k^4}{x^4}$$

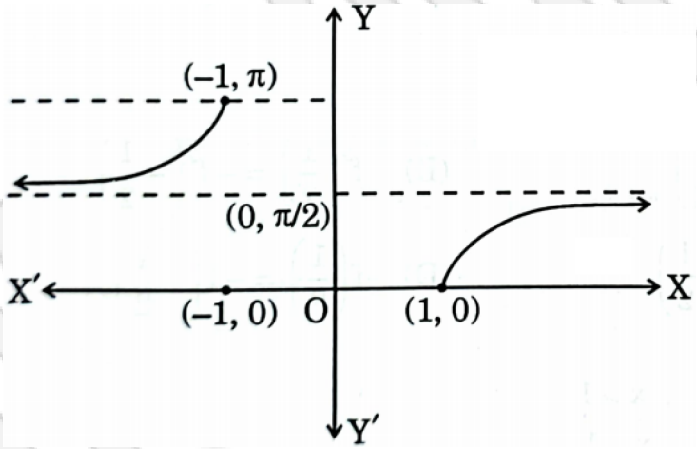
That is, the general solution is $x^2(x^2+2y^2)=C$, where $C=k^4$.

Further it is given that $y(1)=1$. So, $1^2(1^2+2\times1^2)=C \Rightarrow C=3$.

Hence, the solution of D.E is given by $x^2(x^2+2y^2)=3$.

☑ Q.P. Code - 65/7/3

01. The given graph illustrates



(a) $y=\sec^{-1}x$ (b) $y=\cot^{-1}x$ (c) $y=\tan^{-1}x$ (d) $y=\text{cosec}^{-1}x$

Sol. (a) The graph illustrates $y=\sec^{-1}x$.

08. If $f(x)=\begin{cases}3ax-b, & x>1\\ 11, & x=1\\ -5ax-2b, & x<1\end{cases}$ is continuous at $x=1$, then the values of a and b are

(a) $a=3,\ b=5$ (b) $a=8,\ b=-1$ (c) $a=1,\ b=-8$ (d) $a=-3,\ b=5$

Sol. (c) As f is continuous at $x=1$ so, $\lim\limits_{x\to1^-}f(x)=\lim\limits_{x\to1^+}f(x)=f(1)$.

Then $\lim\limits_{x\to1^-}(-5ax-2b)=\lim\limits_{x\to1^+}(3ax-b)=11 \Rightarrow(-5a-2b)=(3a-b)=11$

Consider $3a-b=11,\ -5a-2b=11$

On solving these equations, we get $a=1,\ b=-8$.

10. Edge of a variable cube increases at the rate of 5 cm/s. The rate at which the surface area of the cube increases when the edge is 2 cm long is

(a) $24\ cm^2/s$ (b) $120\ cm^2/s$ (c) $12\ cm^2/s$ (d) $5\ cm^2/s$

Sol. (b) Given $\frac{da}{dt}=5$ cm/s, where a be the edge length of cube.

Surface area of cube, $S=6a^2$

$\Rightarrow \frac{dS}{dt}=12a\times\frac{da}{dt}$

When $a=2$ cm, then $\frac{dS}{dt}=12\times2\times5=120\ cm^2/s$.

14. $\int\frac{e^{-x}}{16+9e^{-2x}}dx$ is equal to

(a) $\frac{16}{9}\tan^{-1}(e^{-x})+C$ (b) $-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right)+C$

(c) $\tan^{-1}\left(\frac{e^{-x}}{4}\right)+C$ (d) $-\frac{1}{3}\tan^{-1}\left(\frac{e^{-x}}{4}\right)+C$

Sol. (b) Put $e^{-x}=t\Rightarrow e^{-x}dx=-dt$.

So, $\int\frac{e^{-x}}{16+9e^{-2x}}dx=\int\frac{-1}{16+9t^2}dt=-\frac{1}{4}\times\frac{1}{3}\tan^{-1}\left(\frac{3t}{4}\right)+C=-\frac{1}{12}\tan^{-1}\left(\frac{3e^{-x}}{4}\right)+C$.

17. If A and B are two events such that $P(B)=\frac{1}{5}$, $P(A\mid B)=\frac{2}{3}$ and $P(A\cup B)=\frac{3}{5}$, then P(A) is

(a) $\frac{10}{15}$ (b) $\frac{2}{15}$ (c) $\frac{1}{5}$ (d) $\frac{8}{15}$

Sol. (d) As $P(A\cap B)=P(A\mid B)\times P(B)=\frac{2}{3}\times\frac{1}{5}=\frac{2}{15}$.

Now $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\Rightarrow\frac{3}{5}=P(A)+\frac{1}{5}-\frac{2}{15}$

$\therefore P(A)=\frac{2}{5}+\frac{2}{15}=\frac{8}{15}$.

22. If $A=\begin{bmatrix}1&0\\-1&5\end{bmatrix}$, then find the value of K if $A^2=6A+KI_2$, where I_2 is an identity matrix.

Sol. $A^2=\begin{bmatrix}1&0\\-1&5\end{bmatrix}\begin{bmatrix}1&0\\-1&5\end{bmatrix}=\begin{bmatrix}1&0\\-6&25\end{bmatrix}$, $6A=6\begin{bmatrix}1&0\\-1&5\end{bmatrix}=\begin{bmatrix}6&0\\-6&30\end{bmatrix}$, $KI_2=\begin{bmatrix}K&0\\0&K\end{bmatrix}$.

Given that $A^2=6A+KI_2$ so, $\begin{bmatrix}1&0\\-6&25\end{bmatrix}=\begin{bmatrix}6&0\\-6&30\end{bmatrix}+\begin{bmatrix}K&0\\0&K\end{bmatrix}$

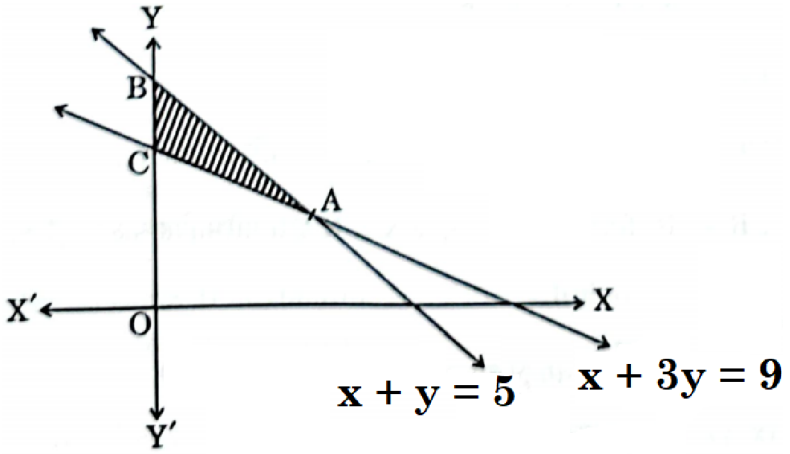
$\Rightarrow\begin{bmatrix}1&0\\-6&25\end{bmatrix}=\begin{bmatrix}6+K&0\\-6&30+K\end{bmatrix}$

On comparing the corresponding terms, we get $1=6+K$, $25=30+K$

$\Rightarrow K=1-6=-5$; $K=25-30=-5$

$\therefore K=-5$.

24. For a Linear Programming Problem, find min $Z=5x+3y$ (where Z is the objective function) for the feasible region shaded in the given figure.



(Note : The figure is not to scale.)

Sol. The corner points are A(3, 2), B(0, 5) and C(0, 3).
Now $Z_A = 5(3)+3(2) = 21$, $Z_B = 5(0)+3(5) = 15$, $Z_C = 5(0)+3(3) = 9$.
Hence, minimum value of Z is 9 and it is obtained at (0, 3).

28. Differentiate $\log(x^x + \text{cosec}^2 x)$ with respect to x.

Sol. Let $u = x^x = e^{\log x^x}$

$\Rightarrow u = e^{x\log x}$

$\Rightarrow \dfrac{du}{dx} = e^{x\log x}\left(x\times\dfrac{1}{x}+\log x\times 1\right) = x^x(1+\log x)$

That is, $\dfrac{d(x^x)}{dx} = x^x(1+\log x)$ …(i)

Now $\dfrac{d}{dx}\left[\log(x^x+\text{cosec}^2 x)\right] = \dfrac{1}{(x^x+\text{cosec}^2 x)}\times\dfrac{d}{dx}(x^x+\text{cosec}^2 x)$

$= \dfrac{1}{(x^x+\text{cosec}^2 x)}\times\left[\dfrac{d}{dx}(x^x)+2\,\text{cosec}\,x(-\text{cosec}\,x\cot x)\right]$

$= \dfrac{1}{(x^x+\text{cosec}^2 x)}\times\left[x^x(1+\log x)-2\,\text{cosec}^2 x\cot x\right]$ [By (i)

29. Show that of all the rectangles with a fixed perimeter, the square has the greatest area.

Sol. Let length and breadth of the rectangle be x and y (in units) respectively.
Then $2(x+y) = P$ …(i)

Area of rectangle, $A = xy = x\left(\dfrac{P}{2}-x\right) = \dfrac{Px}{2}-x^2$

Now $\dfrac{dA}{dx} = \dfrac{P}{2}-2x$ and $\dfrac{d^2A}{dx^2} = -2$

For $\dfrac{dA}{dx} = 0$, $\dfrac{P}{2}-2x = 0$ $\quad\therefore x = \dfrac{P}{4}$

As $\left(\dfrac{d^2A}{dx^2}\right)_{\text{at } x=\frac{P}{4}} = -2 < 0$ so, A is greatest at $x = \dfrac{P}{4}$.

By (i), $y = \dfrac{P}{2}-x = \dfrac{P}{2}-\dfrac{P}{4} = \dfrac{P}{4}$.

Therefore, the length and breadth of rectangle having fixed perimeter and greatest area are both same; that implies the rectangle is definitely a square.

32. (a) Find $\int \frac{3x+1}{(x-2)^2(x+2)}dx$.

OR

(b) Evaluate $\int_0^{\pi/2} \frac{x}{\cos x+\sin x}dx$.

Sol. (a) Consider $\frac{3x+1}{(x-2)^2(x+2)} = \frac{A}{x-2}+\frac{B}{(x-2)^2}+\frac{C}{x+2}$

$\Rightarrow 3x+1 = A(x-2)(x+2)+B(x+2)+C(x-2)^2$

On comparing the coefficients of like terms, we get $A=\frac{5}{16}$, $B=\frac{7}{4}$, $C=-\frac{5}{16}$

Now $\int \frac{3x+1}{(x-2)^2(x+2)}dx = \int\left[\frac{5}{16}\times\frac{1}{x-2}+\frac{7}{4}\times\frac{1}{(x-2)^2}-\frac{5}{16}\times\frac{1}{x+2}\right]dx$

$= \frac{5}{16}\times\log|x-2|-\frac{7}{4}\times\frac{1}{(x-2)}-\frac{5}{16}\times\log|x+2|+C$

$= \frac{5}{16}\times\log\left|\frac{x-2}{x+2}\right|-\frac{7}{4}\times\frac{1}{(x-2)}+C.$

OR

(b) Let $\int_0^{\frac{\pi}{2}} \frac{x}{\cos x+\sin x}dx$ ...(i)

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos\left(\frac{\pi}{2}-x\right)+\sin\left(\frac{\pi}{2}-x\right)}dx$

$\Rightarrow I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}-x}{\cos x+\sin x}dx$ ...(ii)

Adding (i) and (ii), we get $2I = \frac{\pi}{2}\int_0^{\frac{\pi}{2}} \frac{1}{\cos x+\sin x}dx$

$\Rightarrow I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos x+\frac{1}{\sqrt{2}}\sin x\right)}dx$

$\Rightarrow I = \frac{\pi}{4}\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}dx$ $\Rightarrow I = \frac{\pi}{4\sqrt{2}}\int_0^{\frac{\pi}{2}} \sec\left(x-\frac{\pi}{4}\right)dx$

$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\frac{\pi}{2}}$

$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\left[\log\left|\sec\frac{\pi}{4}+\tan\frac{\pi}{4}\right|-\log\left|\sec\left(-\frac{\pi}{4}\right)+\tan\left(-\frac{\pi}{4}\right)\right|\right]$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\left[\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right]$$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\left[\log\frac{\sqrt{2}+1}{\sqrt{2}-1}\right] = \frac{\pi}{4\sqrt{2}}\left[\log\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)\right]$$

$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\left[\log\left(\frac{(\sqrt{2}+1)^2}{2-1}\right)\right] = \frac{\pi}{4\sqrt{2}}\left[2\log(\sqrt{2}+1)\right]$$

Hence, $I = \frac{\pi}{2\sqrt{2}}\left[\log(\sqrt{2}+1)\right]$.

34. Solve the differential equation $(x - \sin y)\,dy + (\tan y)\,dx = 0$, given $y(0) = 0$.

Sol. Rewriting the D.E., we get $\frac{dx}{dy} + (\cot y)\,x = \cos y$

On comparing with $\frac{dx}{dy} + P(y)\,x = Q(y)$, we get $P(y) = \cot y$, $Q(y) = \cos y$.

Integration factor $= e^{\int \cot y\,dy} = e^{\log \sin y} = \sin y$.

The solution of D.E. is given by $(\sin y)\,x = \int \sin y \cos y\,dy + C$

$$\Rightarrow x \sin y = \frac{1}{2}\sin^2 y + C$$

It is given that $y(0) = 0$ i.e., $y = 0$ when $x = 0$.

So, $0 \sin 0 = \frac{1}{2}\sin^2 0 + C \qquad \Rightarrow C = 0$

Hence, the required solution is given by $x \sin y = \frac{1}{2}\sin^2 y$ or, $2x = \sin y$.

With regards,
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